A Review of Nonlinear Eigenvalue Problems

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In its most general form, the nonlinear eigenvalue problem is to find scalars λ (eigenvalues) and nonzero vectors x and y (right and left eigenvectors) satisfying $N(\lambda)x = 0$ and $y^*N(\lambda) = 0$, where $N\colon \Omega \to \mathbb{C}^{n\times n}$ is an analytic function on an open set $\Omega \subseteq \mathbb{C}$. In practice, the matrix elements are most often polynomial, rational or exponential functions of λ or a combination of these. These problems underpin many areas of computational science and engineering. They can be difficult to solve due to large problem size, ill conditioning (which means that the problem is very sensitive to perturbations and hence hard to solve accurately), or simply a lack of good numerical methods. My aim is to review the recent research directions in this area.